Comments on "Energy Considerations for Attitude Stability of Dual-Spin Spacecraft"

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In a recent Note,[1] Prof. Fang obtains criteria for stability of an equilibrium motion of a dual-spin spacecraft by determining whether the equilibrium motion corresponds to the minimum energy state of permissible motions. The equations he employs to determine the minimum energy state treat the dual-spin spacecraft as consisting of a rigid symmetric rotor, body S, which rotates relative to a rigid asymmetric body, body A, in a force-free field. The axis of relative rotation between the two bodies corresponds to the axis of symmetry of the rotor and to a principal axis of body A. It is assumed that no net torque is exerted about this axis between the two bodies.

Although admittedly not modeled in his equations, he states that the analysis is made with the assumption that "the bodies A and S possess some degree of flexibility and some internal dissipative mechanism," and leaves the reader with the incorrect impression that his results are generally valid for this case (possibly excepting "highly flexible spacecraft"). In fact, it is possible to make conclusive statements about the stability of the equilibrium solution only for the particular equations being examined regardless of the method of analysis. Extending results for this case to the same equilibrium solution for other, more complex, equations (multibody or flexible body equations) is at best an intuitive and heuristic procedure and can be only approximate.

Even though this type of analysis is not rigorous, the stability criteria gained thereby are still useful as a "rule of thumb" to guide more exact analyses or spacecraft design choices. Similar forms of this type of analysis have been performed by Iorillo[2] and Likins[3] for dual-spin spacecraft. Fang references the Likins' paper and employs his spacecraft model and, to a large extent, notation. His results, however, differ from those of Likins (and Iorillo, whom Likins has followed). Fang's results are those that would be obtained for energy dissipation on body A only, and agree with those of Likins for this special case only.

The stability of the equilibrium solution

$$\omega_1 = 0, \quad \omega_2 = 0, \quad \omega_3{}^S = \text{const}, \quad \omega_3{}^A = \text{const} \tag{1}$$

is examined. The stability criterion is given by Likins as

$$(P_A/\lambda_A) + (P_S/\lambda_S) < 0 \tag{2}$$

where P_A and P_S represent the average energy dissipation rates on bodies A and S, respectively. Obviously,

$$P_A \leq 0, \qquad P_S \leq 0 \tag{3}$$

If no dissipative mechanism exists on the rotor, $P_S = 0$, and the stability criterion becomes

$$\lambda_A > 0 \tag{4}$$

This criterion is equivalent to requiring that

$$\omega_3{}^A < h_0/I_1 \tag{5}$$

where I_1 is the maximum moment of inertia, and $h_0 = I_3{}^S\omega_3{}^S + I_3{}^A\omega_3{}^A > 0$.

Fang's stability criterion is given as

$$\mu^2 \geq 1 \tag{6}$$

where μ is defined as

$$\mu = I_3{}^S\omega_3{}^S/h(1 - I_3{}^A/I_1) \tag{7}$$

It can be shown that requiring $\mu > 1$ is equivalent to requiring $\omega_3{}^A < h_0/I_1$ by using a similar implied result of Fang,

$$\omega_3{}^A < I_3{}^S\omega_3{}^S/I_1(1 - I_3{}^A/I_1) \tag{8}$$

substituting his equation

$$\omega_3{}^A = (h - I_3{}^S\omega_3{}^S)/I_3{}^A \tag{9}$$

and manipulating. Apparently μ^2 rather than μ is used in the criterion to make the stipulation $h, h_0 > 0$ unnecessary.

Fang asserts that if the criterion is violated, i.e., $\mu^2 < 1$, the equilibrium condition (1) is unstable and the spacecraft moves to the minimum energy state

$$\begin{gathered} \omega_1 = \pm(1/I_1)\{h^2 - [I_3{}^S\omega_3{}^S(t_0)/(1 - I_3{}^A/I_1)]^2\}^{1/2} \\ \omega_2 = 0, \qquad \omega_3{}^S = \omega_3{}^S(t_0) \\ \omega_3{}^A = I_3{}^S\omega_3{}^S(t_0)/(1 - I_3{}^A/I_1)I_1 \end{gathered} \tag{10}$$

which is now the stable equilibrium state. Although Eq. (10) is indeed one of the three possible equilibrium states for his rigid-body model, he is incorrect in asserting that it represents an equilibrium solution for a spacecraft with arbitrary dissipative mechanisms. One cannot, in general, make valid statements about the existence and nature of equilibrium solutions without specifying the particular dissipative mechanism any more than one can make valid statements about stability. In fact, the assumption that Eq. (1) is an equilibrium solution of the spacecraft equations of motion restricts the nature of the allowable dissipative mechanisms. Fang erroneously claims that Eq. (10) is the equilibrium solution of the equations for Likins' first spacecraft model, which has a one-degree-of-freedom spring-mass damper as a dissipative mechanism on body A, with the damper displacement $Z = 0$. Although a neighboring equilibrium solution does exist with $Z \neq 0$, it is not necessarily the minimum energy state if I_1 is not the axis of maximum inertia.

If the dissipative mechanism is confined to body S only, $P_A = 0$, and Likins' criterion yields

$$\lambda_S > 0 \tag{11}$$

for stability. This is equivalent to requiring that

$$\omega_3{}^S < h_0/I_1 \tag{12}$$

The symmetrical results of Eqs. (5) and (12) indicate stability criteria for energy dissipation on either one of the two bodies. This symmetry indicates that the labeling of bodies A and S is arbitrary and is, therefore, not surprising. A result equivalent to Eq. (12) can be obtained using Fang's approach by minimizing the kinetic energy [his Eq. (5)†] with respect to $\omega_3{}^S$ rather than $\omega_3{}^A$,

$$\omega_3{}^S < I_3{}^A\omega_3{}^A/(1 - I_3{}^S/I_1)I_1 \tag{13}$$

Intuitively, this procedure is difficult to justify since Fang uses the assumption of constant angular velocity of the rotor [his Eq. (1)] in minimizing the energy to obtain the previous result. However, Eqs. (8) and (13) are symmetrical and are equivalent to Eqs. (5) and (12) and, hence, the stability criteria for both of the two cases of energy dissipation on one of the two bodies are derivable using Fang's approach. It does not appear, however, that his approach can be used to establish stability criteria for a dual-spin spacecraft having energy dissipation on both bodies.

Received November 15, 1968.

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† A typographical error appears in Eq. (5) in Ref. 1. In the second term, $\omega_3 S$ should read $(\omega_3{}^S)^2$.

References

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[2] Iorillo, A. J., "Nutation Damping Dynamics of Axisymmetric Roto Stabilized Satellites," presented at American Society of Mechanical Engineers Winter Meeting, Chicago, Ill., Nov. 1965.

[3] Likins, P. W., "Attitude Stability Criteria for Dual-Spin Spacecraft," *Journal of Spacecraft and Rockets*, Vol. 4, No. 12, Dec. 1967, pp. 1638–1643.

Reply by Author to D. L. Mingori and W. J. Russell

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THE method of Ref. 1 is essentially based on the assertion†: "For an isolated spacecraft any disturbance will vanish through internal dissipation and the spacecraft will go to its minimum energy state consistent with the angular momentum and any other explicit constraints." The author agrees with Mingori's clarification that the validity of the stability criterion given in Ref. 1 is subject to the condition of constant absolute rotor spin ($\omega_3{}^S$ = const). The example constructed by Mingori[3] is correct, although by an obvious oversight, the spin axis is taken to be the major axis. However, the commentators' symmetry arguments are not sufficiently persuasive, for a similar stability criterion can be obtained for a spacecraft with a constant relative rotor spin ($\omega_3{}^S - \omega_3{}^A$ = const).[2] The symmetry argument is more relevant for this case, but the criterion can indeed be arranged to exhibit such symmetry. The real challenge is the spacecraft model considered by Mingori in Ref. 4. This model has dampers in both bodies of the spacecraft and, if the author is correct, admits only one constant energy (equilibrium) state, namely a constant spin about the rotor axis. Therefore, for this model, instability can occur only in the following ways:

1) The spacecraft will go into some "limit-cycle like" stationary periodic motion. This motion is possible if during each cycle the energy input is equal to the energy dissipated. Although the existence of such motions is not proved, one feels this has to occur in view of the angular momentum constraint and the existence of dissipation.

2) The spacecraft will not be able to maintain its dual-spin eventually and degenerates into a single-spin vehicle. This can occur if no energy source exists.

Either of the preceding possibilities contradicts the derivations used in Ref. 1. Therefore, the author is inclined to agree with the commentators' statement that the stability criterion given in Ref. 1 excludes possible damping in the rotor.

Russell[5] is correct in saying the minimum energy state given in Eq. (17) of Ref. 1 does not satisfy the equations of motion exactly for the model considered by Likins.[6] As stated in Ref. 1, the exact result can be obtained if the potential energy of deformation of the spring is also taken into consideration. However, it can be shown that, for intentionally introduced damping mechanisms, the potential energy is always small in comparison with the spin energy.[2] The improved accuracy obtained that way is not warranted by the additional algebraic complexity involved and, furthermore, the result will be restricted to a particular damping mechanism. The author cannot agree with Russell's overemphasis on the necessity of postulating specific internal structure in order to discuss the stability of a physical system.

Also in Ref. 5, Russell uses the stability criterion given by Likins[6] to gauge the adequacy of the result of Ref. 1. First of all, Likins himself states his criterion is approximate.[6] Furthermore, if one examines the definitions $P_A = -I_3{}^A\dot{\omega}_3{}^A\lambda_A$ and $P_S = -I_3{}^S\dot{\omega}_3{}^S\lambda_S$ Likins' criterion becomes

$$d/dt(I_3{}^A\omega_3{}^A + I_3{}^S\omega_3{}^S) > 0$$

This is equivalent to saying that, if the spacecraft angular momentum component about the rotor axis always increases, eventually the angular momentum components about other axes would have to vanish and the spacecraft is stable. The statement is logical, but is hardly useful as a stability criterion. To be of use, Likins identifies P_A and P_S as average dissipation rates in body A and body S, respectively. This identification is plausible under the condition stated (bearing friction and torque arbitrarily small). However, the validity of this condition for the cases at hand (constant rotor absolute and relative spin) is doubtful. In all the work referenced, the equations of motion of the rotor are tacitly assumed to be satisfied by a constant rotor spin (absolute or relative). This most likely will require certain motor torques which violate Likins' condition that motor and friction torques be arbitrarily small.

References

[1] Fang, B. T., "Energy Considerations for Attitude Stability of Dual-Spin Spacecraft," *Journal of Spacecraft and Rockets*, Vol. 5, No. 10, Oct. 1968, pp. 1241–1243.

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[3] Mingori, D. L., "Comments on 'Energy Considerations for Attitude Stability of Dual-Spin Spacecraft,' " *Journal of Spacecraft and Rockets*, Vol. 6, No. 3, March 1969, p. 350.

[4] Mingori, D. L., "Effects of Energy Dissipation on the Attitude Stability of Dual-Spin Spacecraft," *AIAA Journal*, Vol. 7, No. 1, Jan. 1969, pp. 20–27.

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Received November 18, 1968; revision received January 2, 1968.

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† A thorough discussion of explicit assumptions can be found in Ref. 2.